the **Further Mathematics** network

Warm up!

Travelling at an average speed of 100km/hr, a train took 3 hours to travel to Birmingham. Unfortunately the train waited just outside the station, which reduced the average speed for the whole journey to 90km/hr. For how many minutes was the train waiting?

A 1 B 5 C 10 D 15 E 20

Question courtesy of UKMT

the **Further Mathematics** network

www.fmnetwork.org.uk

STEP Mathematics Online Course

1. Division by Zero

Let Maths take you Further…

department for children, schools and families

MEI

Mathematics in Education and Industry

What will you learn in this tutorial?

When handling equations care must be taken not to lose any roots when cancelling factors.

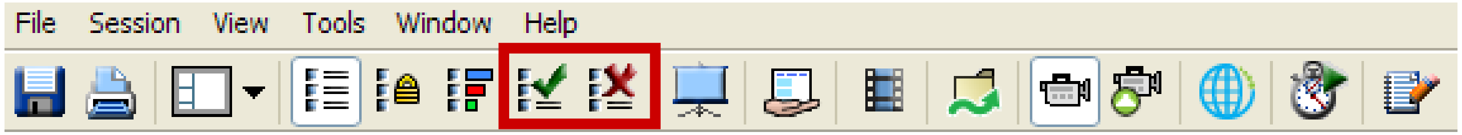
More generally care must be taken to avoid division by zero.

We'll begin by looking at two specific examples in this area:

- The flaw in a proof that 1 = 2.
- Solving $\sin\theta = \sin 2\theta$

A proof that 1 = 2

Here we will use the voting buttons



Step 1: Let $a = b.$

Step 2: Then $a^2 = ab.$

Step 3: So $a^2 + a^2 = a^2 + ab.$

Step 4: In other words $2a^2 = a^2 + ab.$

Step 5: So $2a^2 – 2ab = a^2 + ab – 2ab$

Step 6: and $2a^2 – 2ab = a^2 – ab.$

Step 7: In other words $2(a^2 – ab) = a^2 – ab.$

Step 8: Cancelling the $(a^2 – ab)$ from both sides gives 1 = 2.

Solving **sinθ =sin2θ**

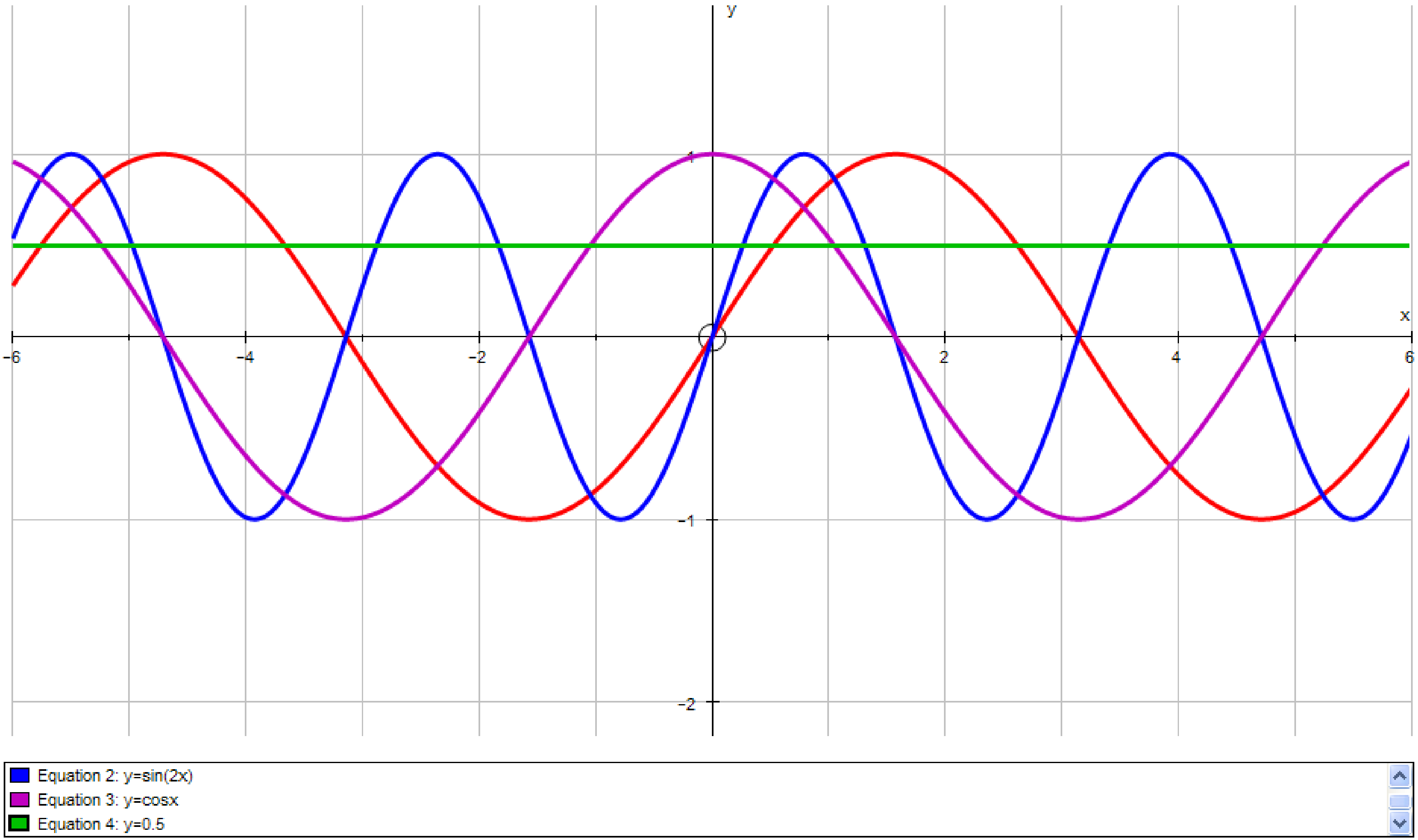
Criticise the following:

If	$\sin\theta = \sin 2\theta$.
Then	$\sin\theta = 2\sin\theta\cos\theta$.
Cancelling $\sin\theta$ gives	$1 = 2\cos\theta$.

So $\sin\theta = \sin 2\theta$ precisely when $\cos\theta = 0.5$

Autograph Demonstration

(see teachers' notes)



Correct procedure when dealing with this situation in equations

Incorrect	Correct
If ba = bc Then, by cancelling b, a = c.	If ba = bc Then ba – bc = 0 So b(a – c) = 0 So either b = 0 or (a – c) =0. So either b = 0 or a = c

Example from this week's problem sheet for discussion

STEP Mathematics I Summer 2003

2 The first question on an examination paper is:

$$\text{Solve for } x \text{ the equation} \quad \frac{1}{x} = \frac{1}{a} + \frac{1}{b}\,.$$

where (in the question) a and b are given non-zero real numbers. One candidate writes $x = a+b$ as the solution. Show that there are no values of a and b for which this will give the correct answer.

The next question on the examination paper is:

$$\text{Solve for } x \text{ the equation} \quad \frac{1}{x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\,.$$

where (in the question) a, b and c are given non-zero numbers. The candidate uses the same technique, giving the answer as $x = a+b+c$. Show that the candidate's answer will be correct if and only if a, b and c satisfy at least one of the equations $a+b=0$, $b+c=0$ or $c+a=0$.

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Questions from last week's problem sheet

The Further Mathematics network

Questions for STEP Tutorial 0 – Introduction to STEP

STEP Mathematics I Summer 2004

1 (i) Express $(3+2\sqrt{5})^2$ in the form $a+b\sqrt{5}$ where a and b are integers.

(ii) Find the positive integers c and d such that $\sqrt[4]{99-70\sqrt{2}} = c-d\sqrt{2}$.

(iii) Find the two real solutions of $x^4-198x^2+1=0$.

STEP Mathematics II Summer 2004

1 Find all real values of x that satisfy:

(i) $\sqrt{3x^2+1}+\sqrt{x}-2x-1=0$;

(ii) $\sqrt{3x^2+1}-2\sqrt{x}+x-1=0$;

(iii) $\sqrt{3x^2+1}-2\sqrt{x}-x+1=0$.

STEP Mathematics II Summer 2006

5 The notation $\lfloor x \rfloor$ denotes the greatest integer less than or equal to the real number x. Thus, for example, $\lfloor \pi \rfloor = 3$, $\lfloor 18 \rfloor = 18$ and $\lfloor -4.2 \rfloor = -5$.

(i) Two curves are given by $y=x^2+3x-1$ and $y=x^2+3\lfloor x \rfloor -1$. Sketch the curves, for $1 \leqslant x \leqslant 3$, on the same axes.

Find the area between the two curves for $1 \leqslant x \leqslant n$, where n is a positive integer.

(ii) Two curves are given by $y=x^2+3x-1$ and $y=\lfloor x \rfloor^2+3\lfloor x \rfloor -1$. Sketch the curves, for $1 \leqslant x \leqslant 3$, on the same axes.

Show that the area between the two curves for $1 \leqslant x \leqslant n$, where n is a positive integer, is

$$\tfrac{1}{6}(n-1)(3n+11).$$

STEP Mathematics I Summer 2004

1 (i) Express $(3+2\sqrt{5})^3$ in the form $a+b\sqrt{5}$ where a and b are integers.

(ii) Find the positive integers c and d such that $\sqrt[3]{99-70\sqrt{2}} = c - d\sqrt{2}$.

(iii) Find the two real solutions of $x^6 - 198x^3 + 1 = 0$.

STEP
Mathematics II
Summer 2004

1 Find all real values of x that satisfy:

(i) $\sqrt{3x^2+1}+\sqrt{x}-2x-1=0\,;$

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