Vectors

What you should know

How to find the equation of a line in the form $y = mx + c$ (this is known as the cartesian equation) given the coordinates of any two points on the line.

The position vector of the point A, with coordinates (a_1, a_2) is $\overrightarrow{\mathrm{A}} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$.

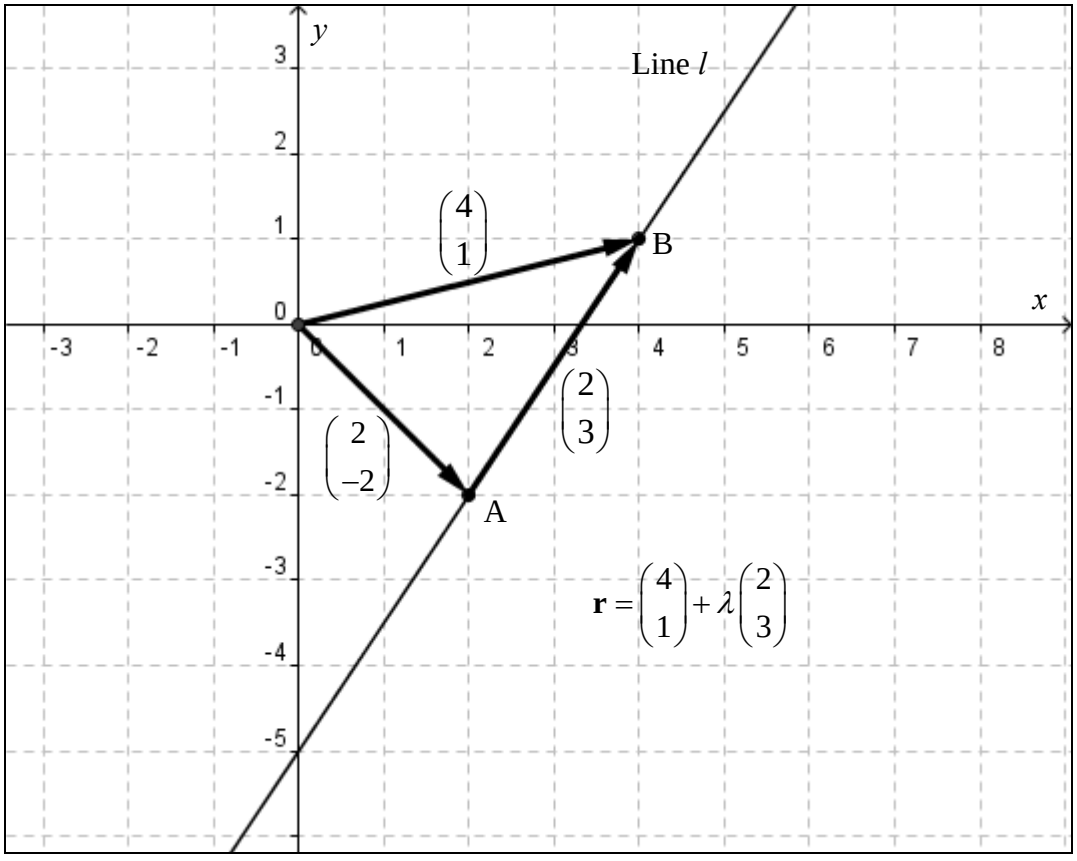
The vector from point A, with position vector $\overrightarrow{\mathrm{A}} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, to point B, with position vector $\overrightarrow{\mathrm{B}} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, is $\overrightarrow{\mathrm{B}} - \overrightarrow{\mathrm{A}} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \end{pmatrix}$.

New ideas

You can use vectors, rather than a cartesian equation, to describe a line. A line is defined uniquely by any two points on it.

The vector between any two points on a line is parallel to that line.

The position vector of any particular point on a line, plus a multiple of any vector parallel to that line, must give the position vector of another point on that line.
It helps to visualise this!



Vectors

Task: The vector equation of a line in two dimensions

The line l that passes through the points A(2, –2) and B(4, 1) is parallel to vector

$\vec{B} - \vec{A} = \begin{pmatrix} 4-2 \\ 1-(-2) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

The position vector $\vec{A} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ gives a point on the line.

The position vector, $\vec{R}$, of any point on the line is $\vec{R} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, where λ can be any number and is called a **parameter**.

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is the vector equation of the line.

- If $\lambda = 2$, what point does the vector equation give?
- Find some other points on the line using different values of λ.
- What is the cartesian equation of this line?
- Do the points you found work in the cartesian equation?
- Can you explain why $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is a vector equation of the same line?

Take it further

- Can you find vector equations for other lines?
- Find out more about vector equations.

Where this goes next

At A level vector equations are studied in Core Mathematics and Further Mathematics.