

Pythagoras' theorem

What you should know

Pythagoras' theorem: $c^2 = a^2 + b^2$ where c is the length of the hypotenuse of a triangle and a and b are the lengths of the shorter sides of the triangle.

How to calculate the areas of triangles and squares.

The meaning of 'congruent'.

How to manipulate algebraic expressions.

New idea

Proving a mathematical idea is true means producing a logical argument that shows it is true without having to test every possible case. For example, you know Pythagoras' theorem always seems to work but there are an infinite number of possible right-angled triangles so it is impossible to check them all. To be sure it is always true you need to prove it mathematically, using an argument that applies to **all** right-angled triangles.

Task: Proof

- Look at diagram 1 on the separate sheet.
It forms the basis for a proof of Pythagoras' theorem.
In the top part of the diagram, four congruent right-angled triangles are arranged to form a large square, with a smaller square inside left uncovered.
How do you know these shapes are both squares?
In the bottom part of the diagram, the same four congruent right-angled triangles are arranged differently, forming an identical large square, but leaving two different, smaller squares uncovered.
By considering both parts of the diagram it is possible to construct an argument that shows that Pythagoras' theorem is true for **all** right-angled triangles.
Try it, then compare your argument with the solution (diagram 1a) on a separate sheet.
- Now look at diagram 2 on the separate sheet.
Use it to construct an alternative proof of Pythagoras' theorem, by considering two ways to represent the area of the diagram algebraically.
- All odd numbers are of the form $2n \pm 1$, where n is an integer. Can you explain why? The difference between consecutive square numbers is always odd. For example, $4^2 - 3^2 = 7$; $10^2 - 9^2 = 19$. Can you use the definition of an odd number to prove this?

Take it further

- Investigate more proofs of Pythagoras' theorem.
- Find out more about different types of proof – some examples are deduction, contradiction, exhaustion, induction and disproof by counter-example.
Are You Sure?, a book published by the Mathematical Association has many examples. (Your school library or mathematics department may have a copy.)
- Find out about Euclid.

Where this goes next

Proof is an important element of A level Mathematics and Further Mathematics. At university level proof underpins all of the ideas covered in a mathematics degree.

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Diagram 1

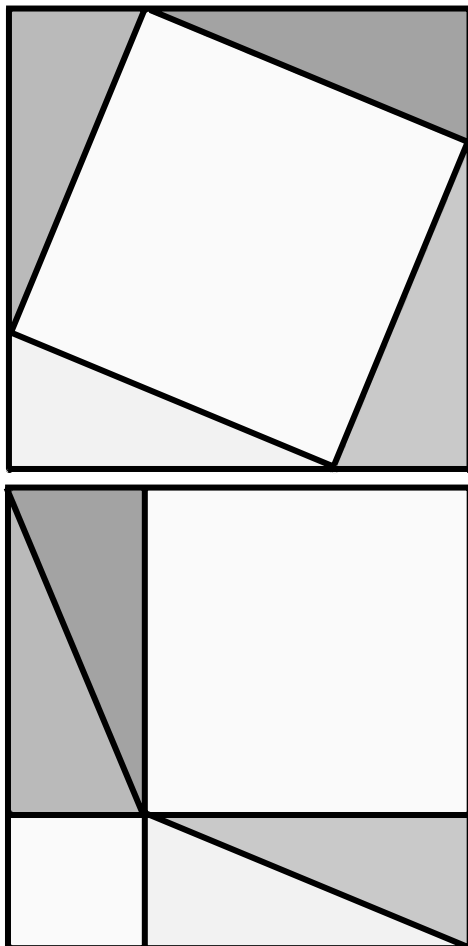


Diagram 2

