Trigonometry 2

What you should know

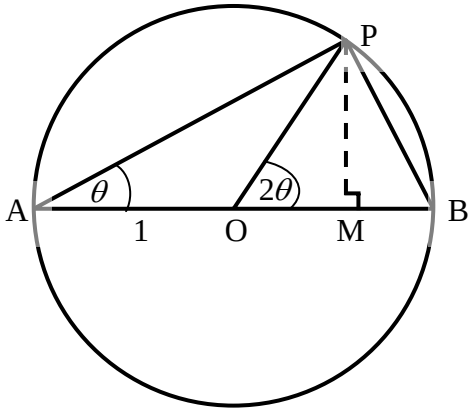
The angles in a triangle add up to 180°.
How to find sides and angles in a triangle using sine and cosine.
The circle theorems.

New idea

It is possible to use the values of sin 13° and cos 13° to work out the values of sin 26° and cos 26°. To do this you need to use the double-angle formulae. In this worksheet you will discover these formulae yourself.

Task: The double-angle formulae

The diagram shows a circle with centre O and radius 1.
AB is a diameter of the circle and P is a point on the circle.
The angle PAB is θ.



- Explain why angle APB = 90° and why angle POB = 2θ.
- Explain why AP = 2 cos θ.
- Copy the diagram. Look at triangle POM.
  Write on your diagram the lengths of the three sides in terms of the angle 2θ.
  You will need to use some trigonometry.
- Now try to find as many other edge lengths as you can. You might want to draw copies of the right-angled triangles APM, PBM and ABP. Your aim is to find expressions for the lengths PM and OM which are different from those you found earlier.
- You should have found that PM = sin 2θ and PM = 2 sin θ cos θ.
  Now, using your calculator, check that these are the same for a few different values of θ. Then do the same for OM. **NB:** sin 2θ means sin (2θ).

Take it further

- You might know these exact values: sin 30° = $\frac{1}{2}$ and cos 30° = $\frac{\sqrt{3}}{2}$.
  Can you use these to find sin 60° and cos 60°?
  How about sin 15° and cos 15°?
- The diagram looks like 2θ has to be less than 90°.
  What happens for larger angles?
- Starting with the graphs of y = sin θ and y = cos θ, sketch the graphs of $y = \sin^2\theta$, $y = \cos^2\theta$, y = sin θ cos θ, y = sin 2θ and y = cos 2θ.
  Explain any links you notice. **NB:** $\sin^2\theta$ means $(\sin\theta)^2$.
- Find a formula for tan 2θ in terms of tan θ.

Where this goes next

At A level you will discover more relationships between sine, cosine and tangent in Core Mathematics.