

Trigonometry 1

What you should know

The trigonometric ratios: $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

Pythagoras' theorem: $\text{opposite}^2 + \text{adjacent}^2 = \text{hypotenuse}^2$

New idea

If you divide $\sin \theta$ by $\cos \theta$ you get

$$\frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{hyp}} \div \frac{\text{adj}}{\text{hyp}}$$

= ...

$$= \tan \theta$$

$$\text{so } \tan \theta = \frac{\sin \theta}{\cos \theta}.$$

$\sin^2 \theta$ means $(\sin \theta)^2$ so you can write

$$\sin^2 \theta = \left(\frac{\text{opp}}{\text{hyp}} \right)^2.$$

Using this idea you can add together $\sin^2 \theta$ and $\cos^2 \theta$ to get

$$\sin^2 \theta + \cos^2 \theta = \left(\frac{\text{opp}}{\text{hyp}} \right)^2 + \left(\frac{\text{adj}}{\text{hyp}} \right)^2$$

= ...

$$= 1$$

$$\text{so } \sin^2 \theta + \cos^2 \theta = 1.$$

Task: Trigonometric identities

- In both of the ideas above, what should the missing bits of algebra, represented by ..., be? (There is more than one line of algebra missing!)
- Find the sine and cosine of some angles and check that, in every case, $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$.
Remember: $\sin^2 \theta$ means $(\sin \theta)^2$ and $\cos^2 \theta$ means $(\cos \theta)^2$.

Take it further

- You can use these identities to find values for $\cos \theta$ and $\tan \theta$ even if you only know $\sin \theta$. You don't even need to find out what the angle θ is.
Try finding $\cos \theta$ and $\tan \theta$ when $\sin \theta = \frac{3}{5}$.
- Can you find some other right-angled triangles where $\sin \theta$, $\cos \theta$ and $\tan \theta$ are all rational numbers (fractions that can be written as one integer over another)?
- Do these identities work when $\theta < 0^\circ$ or $\theta > 90^\circ$?
- Find out what **secant**, **cosecant** and **cotangent** are.
- Try dividing $\sin^2 \theta + \cos^2 \theta = 1$ by $\cos^2 \theta$. What identity do you get?

Where this goes next

At A level trigonometry is studied in Core Mathematics.