Solving quadratic equations

What you should know

How to solve a quadratic equation $ax^2 + bx + c = 0$ using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

New idea

When the number under the square root is negative you usually stop because "you can't square root a negative number" … but what if you could?

If you wanted to find a number that was the square root of -1 it couldn't be positive, negative or zero. (Why not?) In fact it can't be any 'real' number, so it must be an 'imaginary' number. The square root of -1 is called i.

So $\sqrt{-1} = i$ and $i^2 = -1$

Task: Complex numbers

If you solve the quadratic equation

$x^2 - 4x + 5 = 0$

using the formula, you get

$$x = \frac{4 \pm \sqrt{-4}}{2}$$

so one of the roots is

$$x = \frac{4 + \sqrt{4}\sqrt{-1}}{2}$$

$$= \frac{4 + 2i}{2} = 2 + i$$

This value of x works in the equation!

Check:

$(2 + i)^2 - 4(2 + i) + 5$

$= 4 + 2i + 2i - 1 - 8 - 4i + 5$

$= 0$

Numbers like $2 + i$ that have a **real** and an **imaginary** part are called **complex numbers**.

- Why does $(2 + i)^2$ expand to $4 + 2i + 2i - 1$?
- What is the other root of $x^2 - 4x + 5 = 0$?
  Does this root work in the equation?
- Can you find other quadratic equations that have a negative number under the square root?
  Can you solve these equations?

Take it further

- What do the graphs of this type of quadratic equation look like?
- Find out more about complex numbers.

Where this goes next

At A level complex numbers are studied in Further Mathematics.