

Simultaneous equations

What you should know

How to solve a pair of simultaneous equations in two unknowns by substitution.

For example: $2x + 3y = 5$
 $x + y = 2$

New idea

If you have three equations in three unknowns you can solve them by using substitution twice.

Task: Simultaneous equations in three unknowns**Example**

Solve these simultaneous equations.

$$3x + 4y + z = 3 \quad (1)$$

$$x + y + z = 2 \quad (2)$$

$$2x + y - z = 2 \quad (3)$$

Rewrite equation (3) as $2x + y - 2 = z$ and substitute this into the first two equations.

$$(1) \quad 3x + 4y + 2x + y - 2 = 3$$

which simplifies to

$$5x + 5y = 5 \text{ or } x + y = 1 \quad (4)$$

$$(2) \quad x + y + 2x + y - 2 = 2$$

which simplifies to $3x + 2y = 4 \quad (5)$

Equation (4) can be rewritten as

$$y = 1 - x.$$

Substituting $y = 1 - x$ into equation (5) gives $3x + 2(1 - x)$ which simplifies to $x + 2 = 4$, so $x = 2$.

Substituting $x = 2$ into $y = 1 - x$ gives $y = -1$.

Substituting $x = 2$ and $y = -1$ into $2x + y - 2 = z$ gives $z = 1$.

You can check these three values by substituting them into the three original equations.

- Make up values for x , y and z . For example, $x = 3$, $y = 1$ and $z = -4$.
- Make up three expressions in x , y and z . For example, $x + y + z$, $2x + 3y - z$ and $3x - y + z$.
- It is important that your expressions are not the same as or multiples of each other, such as $x + y + z$ and $2x + 2y + 2z$.
- Work out what your expressions equal when you substitute in your values for x , y and z .
e.g. $x + y + z = 0$
 $2x + 3y - z = 13$
 $3x - y + z = 4$
- Give these equations to another student and ask them to find your values for x , y and z .

Take it further

- In the task, it was important that your expressions were not the same or multiples of each other. Can you explain why?
- Investigate what a set of three simultaneous equations look like using a 3D graph plotter.

Where this goes next

At A level techniques for dealing with lots of simultaneous equations with many variables involve vectors and matrices which are covered in Core Mathematics and Further Mathematics.