

A1


Five perfect squares have a mean of 59, a median of 4 and a mode of 1.

The second largest of the five numbers is a two digit number ' ab '.

What is the value of $a - b$?



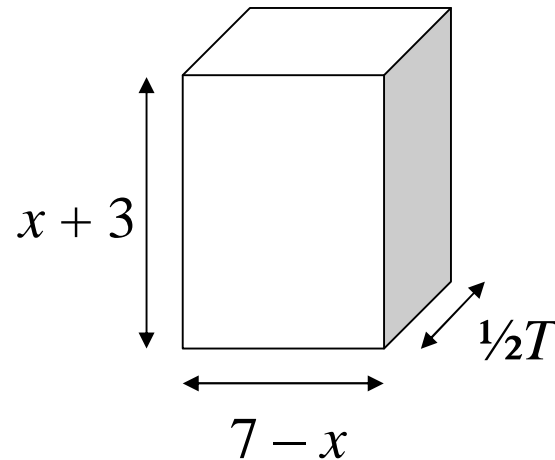
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A2 *T is the number that you will receive.*

Calculate the maximum possible volume of the cuboid shown, as x varies.



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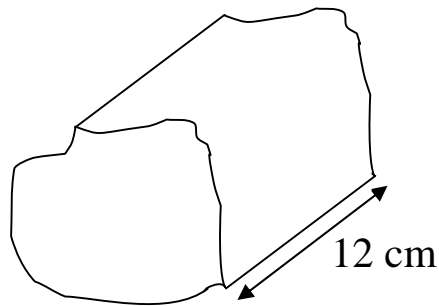
A3 *T is the number that you will receive.*

Prism A has a cross-sectional area of $T \text{ cm}^2$.

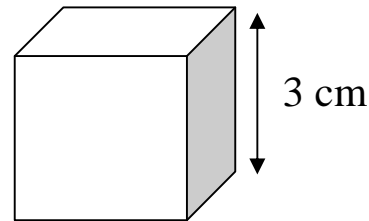
Cuboid B has the same volume as prism A.

Cuboid B has a square base and a height of 3 cm.

Calculate the surface area of the cuboid.




Prism A



Cuboid B



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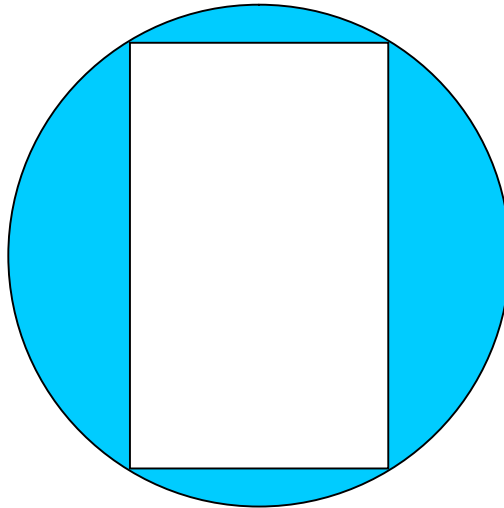


A4 *T is the number that you will receive.*


$T = 32d$ where d is the diameter of the circle shown.

The rectangle has a height equal to twice its width.

Calculate the shaded area, giving your answer in terms of π .



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B1


The function $f(y)$ is defined for values of y , ($y \neq 0$ or 1), by

$$f(y) = \frac{1 + \frac{1}{y}}{1 - \frac{1}{y}}$$

Calculate $f(f(f(6)))$ and write your answer in the form $\frac{a}{b}$, where a and b are positive integers with no common factors except 1, and then find the value of $a + b$.



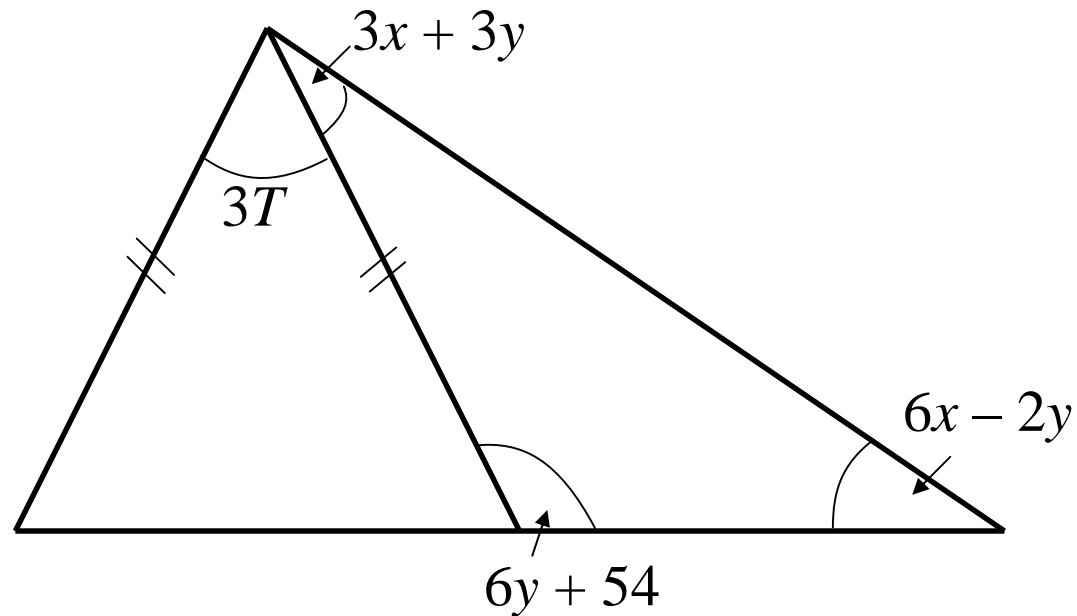
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B2 *T is the number that you will receive.*

Find the value of x .



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B3 *T is the number that you will receive.*

Find the smallest positive multiple of T which leaves a remainder of 1 upon division by each of the numbers 2, 3, 4, 5 or 6.



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B4 *T is the number that you will receive.*

For any positive integer y , we define $y! = 1 \times 2 \times 3 \times 4 \times 5 \times \dots \times y$.

Write down the number of zeros at the end of $\left(\frac{T-1}{10}\right)!$



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C1

A quadrilateral has angles such that, when put in order of increasing size, they form a sequence where each term is twice the previous term. Find the size of the second smallest angle.



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C2 *T is the number that you will receive.*

If $\left(\frac{T}{6}\right)^{3y+1} = 16^{y+1}$, find the value of $\frac{1}{y}$.



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C3 *T is the number that you will receive.*

The points with co-ordinates $(0,0)$, $(T, 1)$ and $(1,T)$ are the vertices of a triangle.

Find the area of the triangle.



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C4 *T is the number that you will receive.*

Lance cycles up a hill at an average speed of 4 m/s and then back via the same route at an average speed of T m/s. Find his average speed for the whole journey.



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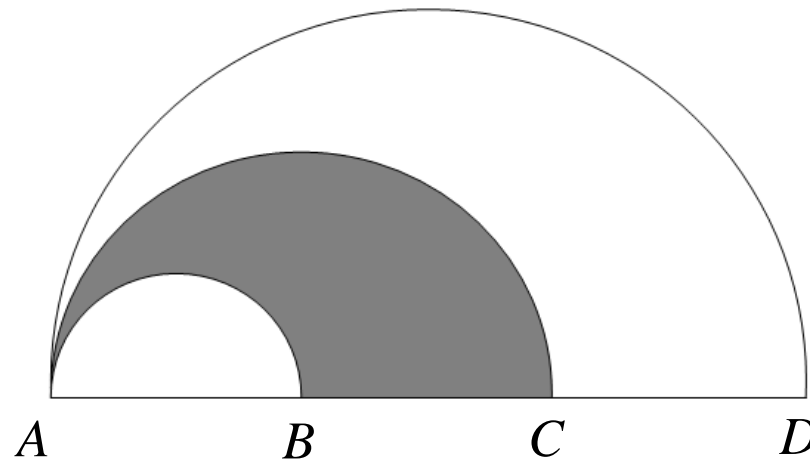
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D1

The diagram shows three semicircles, and $AB = BC = CD$.

What fraction of the largest semicircle is shaded?



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D2 *T is the number that you will receive.*


The value of the expression below can be written as a fraction in the form $\frac{a}{b}$, where a and b are positive integers with no common factors except 1.

$$1 + \frac{T}{1 + \frac{T+1}{1 + \frac{1}{1+1}}}$$

Calculate the value of $a - b$.

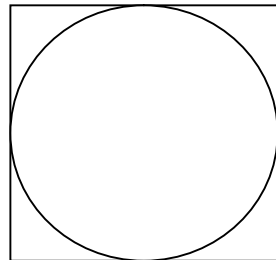


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D3 *T is the number that you will receive.*



The area of the circle above is $(T + 13) \text{ cm}^2$. If the area of the square is written in the form $\frac{m}{\pi}$, what is the value of m ?



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D4 *T is the number that you will receive.*

Suppose x and y are positive real numbers where


$$x(x + y) = T - 60$$

and $y(x + y) = T - 55$

Calculate the value of $x + y$, giving your answer as a surd.



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E1


If $x = \sqrt{5}$ and $y = \frac{x - \frac{1}{x}}{x + \frac{1}{x}}$, find the value of $\frac{\frac{1}{y} - y}{\frac{1}{y} + y}$ and express your answer

as a fraction in the form $\frac{a}{b}$, where a and b are positive integers with no common factors except 1.

Calculate the value of $b - a$.



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E2 *T is the number that you will receive.*

By rationalising the denominator, express the fraction $\frac{T\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ in the

form $a+b\sqrt{c}$, for suitable integers a , b and c . What is the value of $\frac{a+b}{c+6}$?



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E3 *T is the number that you will receive.*

When the expression $(T - \sqrt{2})^3 + (T + \sqrt{2})^2 + (T - \sqrt{2})$ is simplified, it can be expressed in the form $a - b\sqrt{c}$. What is the value of $a - bc$?



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E4 *T is the number that you will receive.*

Define the operation Δ by $a \Delta b = ab - 2a - 2b + 6$.

What is the value of $(a \Delta b) \Delta c - a \Delta (b \Delta c) + 2T$?



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F1

For any triangle, the incircle of a triangle is the largest circle contained inside the triangle; it touches (is tangent to) the three sides.

Find the radius of the incircle for a triangle with sides of length 8 units, 15 units and 17 units.



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F2 *T is the number that you will receive.*

A train leaves London for Edinburgh every hour on the hour. A train leaves Edinburgh for London every hour on the half hour. Each train takes $T + 2$ hours to reach its destination. If you travel on the London to Edinburgh train, how many trains from Edinburgh to London would the driver see on your journey?



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F3 *T is the number that you will receive.*

The system of equations

$$x + y = 1$$


$$kx + y = T - 8$$

$$x + ky = T - 7$$

has a unique solution. Find the value of $\frac{k}{2}$.



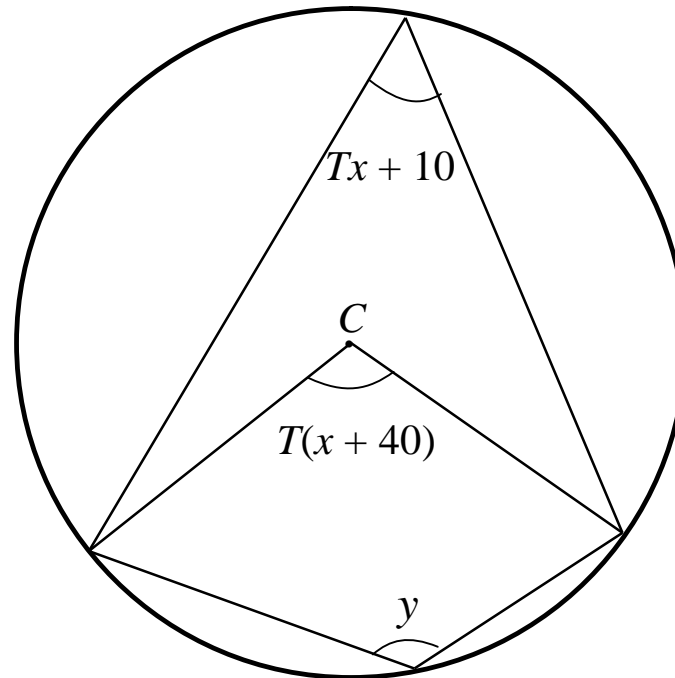
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F4 *T is the number that you will receive.*

For the circle centre C , find the value of y .



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