GROUP ROUND

INSTRUCTIONS

- Your team will have 40 minutes to answer 10 questions. Each team will have the same questions.

- Each question is worth 6 points. However, some questions are easier than others!

- You will have to decide your team's strategy for this group competition. Do you split up so that individuals work on a few questions each or do you work in pairs on a greater number of questions? Working all together on all the questions may well take too long. You decide!

- There is only one answer sheet per team. Five minutes before the end of the time you will be told to finalise your answers and write them on the answer sheet. This answer sheet is the only thing that will be marked.

Question 1

Given that $[f(x)]^2 = x(x+1)(x+2)(x+3)+1$

and $f(0) > 0$

find $f(x)$.

Question 2

Express $\left(2^{1/2}+1\right)\left(2^{1/4}+1\right)\left(2^{1/8}+1\right)\left(2^{1/16}+1\right)\left(2^{1/16}-1\right)$

in its simplest form.

Question 3

Given that

$$\left(x^{2y+1}\right)^{z-1} = \left(x^{2z-1}\right)^{y+1}$$

and

$$\left(y^{z+1}\right)^{2x+1} = \left(y^{z-1}\right)^{2x-1}$$

and that $x \neq 0, 1, -1$ and $y \neq 0, 1, -1,$

find an expression for y in terms of x.

Question 4

The number of ways in which six different books can be shared between two children so that each child receives an odd number of books is 32.

Find the number of ways in which nine different books can be shared amongst three children so that each child receives an odd number of books.

Question 5

Starting with a positive 2-digit number, N, reverse the digits to form another number, M. How many possible values are there for N such that $N - M \geq 0$ and $N - M$ is either a perfect square or a perfect cube?

e.g. $30 - 03 = 27$

Question 6

$$\cfrac{\cfrac{1}{\cfrac{1}{\cfrac{1}{\cfrac{1}{\sqrt{x}}+1}+1}+1}+1}{\cfrac{1}{\cfrac{1}{\cfrac{1}{\cfrac{1}{\sqrt{x}}-1}-1}-1}-1} = 1$$

Calculate x.

Question 7

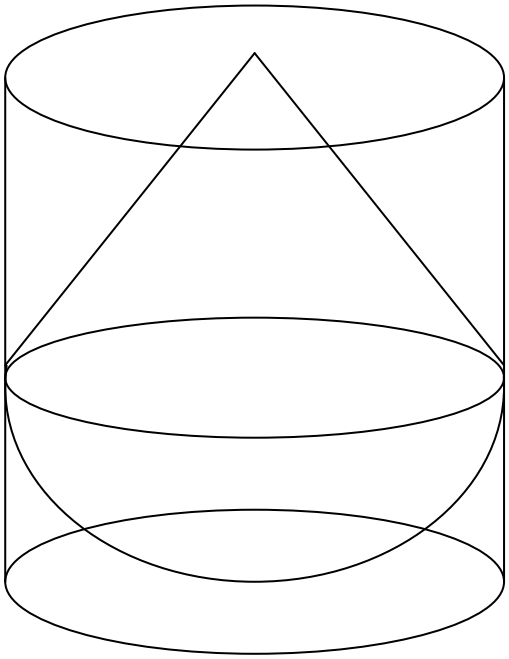
The four terms of this sequence are formed such that each term after the first is the square of the previous term.

$$x-12,\quad \frac{3x-2}{7},\quad \frac{3x+2}{2},\quad ax+6.$$

What is the value of a?

Question 8

A child's wooden toy is made in the shape of a right cone joined to a hemisphere so that their circular faces coincide exactly. The cone and the hemisphere are of equal volume. The toy is packaged in the smallest possible closed cylindrical box.

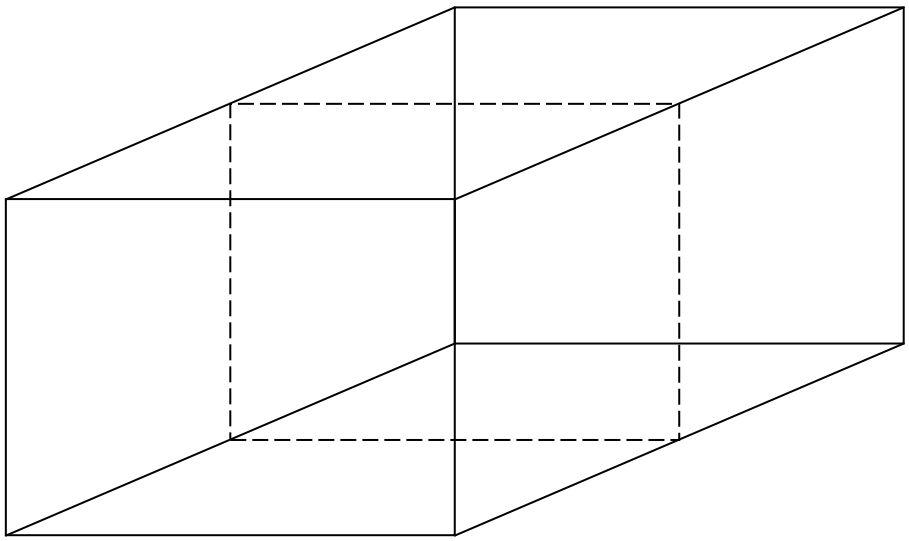


What proportion of the capacity of the box is filled by the toy?

Question 9

This cuboid has the property that when it is cut in half along the plane of symmetry that bisects its longest edges, the resulting halves have edges in the same proportion as the original cuboid.

In what ratio are the sides of the cuboid?

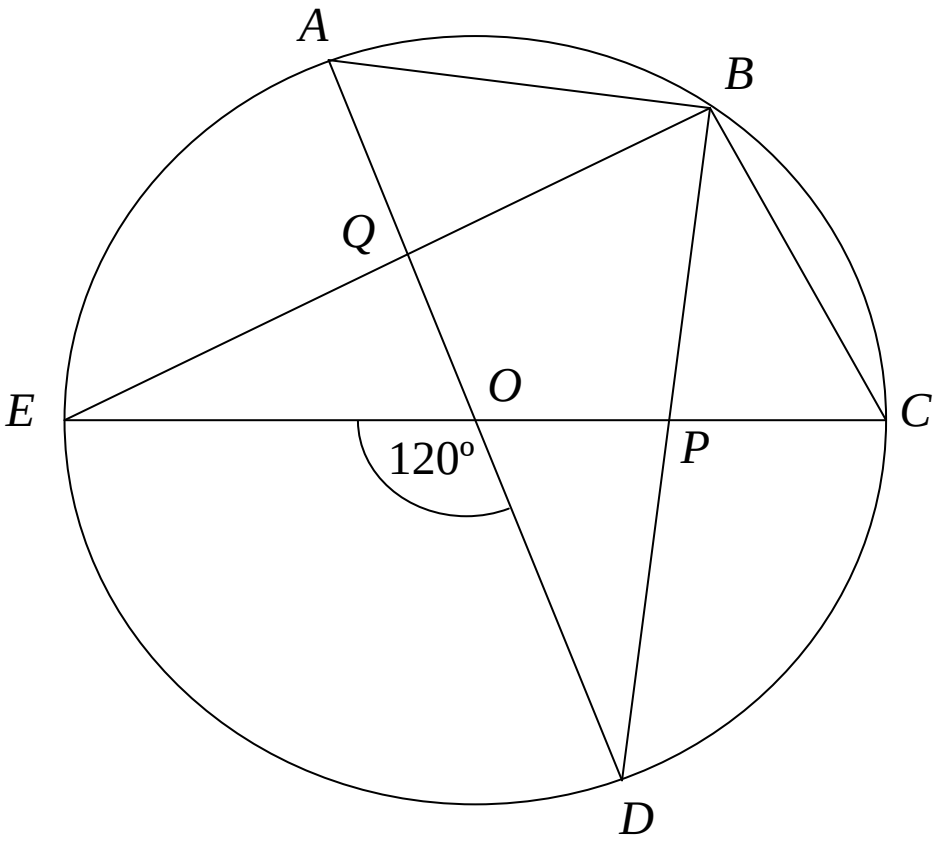


Give your answer in the form $a : b : c$, where $a < b < c$.

Question 10

The points *A*, *B*, *C*, *D* and *E* all lie on a circle with centre *O* and of radius 2 units. The lines *AD* and *CE* are diameters of the circle and the chords *BD* and *BE* are of equal length. Angle *DOE* is 120°.

Calculate the area inside the octagon *ABCPDOEQ*.



Not to scale

Group answer sheet

Team number ……. **Team name ……..……..**

1. $f(x)$ =	**2.** Value of expression =
3. y =	**4.** Number of ways =
5. Number of values for N =	**6.** x =
7. a =	**8.** Proportion filled by the toy =
9. Ratio of sides of the cuboid =	**10.** Area of octagon = units^2

Award 6 points for each correct answer.

TOTAL SCORE = _____

Senior Team Maths Challenge

National Final 2011

Group Round