MEI Conference 2008 Delegate Pack Quiz
ANSWERS

1. 2008! ends in a string of zeros. How many of them are there?

2's and 5's pair off to produce multiples of 10 and since there are more 2's than 5's in the prime factorisation of 2007! we need to find the number of 5's in this factorisation.

2008/5	401 numbers are multiples of 5
2008/25	80 numbers are multiples of 5^2
2008/125	16 numbers are multiples of 5^3
2008/625	3 numbers are multiples of 5^4

401+80+16+3 = 500

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2. 3/07/2008 is a Thursday. On what day of the week will 3/07/3008 fall?

The day advances by one each year except in leap years when it advances by two. Bearing in mind that the only leap years ending in "00" are 2400 and 2800 then we need to consider 1000+(1000/4)-8 =1242 = 7 x177 + 3
Three days after Thursday is Sunday.

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3. What is the maximum possible value of the product of a set of positive integers whose sum is 2008?

The highest product of integers whose sum is 6 is 3 × 3 = 9 which is greater than 2 × 2 × 2 = 8. A little thought shows that the answer is $3^{668} \times 2^2$

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4. The integers from 1 to N are written down in alphabetical order in English. '*Two thousand and eight*' is in position 2008. What is N?.

If N=2008 then 2008 is only seven from the end with 2005, 2004, 2001, 2007, 2006, 2003 and 2002 being the final seven numbers.
So we need to continue until exactly seven more numbers slot in before 2008. These will be 3000 to 3006. So N is 3006.

5. What is the smallest positive integer with exactly 2008 factors?

Since the prime factorisation of 2008 is $2\times2\times2\times251$, the smallest number is $2^{250}\times3\times5\times7$. Note these 2008 factors occur once each in the expansion of

$$\left(1+2+2^2+...+2^{250}\right)(1+3)(1+5)(1+7)$$

and so the sum of all the factors is $\left(2^{251}-1\right)\times4\times6\times8=192\times\left(2^{251}-1\right)\simeq7\times10^{77}$

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6. How many right-angled triangles, with all sides an integer length in centimetres, have shortest side length 2008cm?

$a^2-b^2=2008^2 \quad \Rightarrow \quad (a-b)(a+b)=2^6\times251^2$

$a-b$ and $a+b$ must be both even since their difference is even ($2b$) and the product of two odd numbers is odd.

$a-b$	$a+b$	a	b
2	$2^5\times251^2=2016032$	1008017	1008015
$2^2=4$	$2^4\times251^2=1008016$	504010	504006
$2^3=8$	$2^3\times251^2=504008$	252008	252000
$2^4=16$	$2^2\times251^2=252004$	126010	125994
$2^5=32$	$2\times251^2=126002$	63017	62985
$2\times251=502$	$2^5\times251=8032$	4267	3765
$2^2\times251=1004$	$2^4\times251=4016$	2510	1506

This completes the set since $a+b>a-b$. Notice that the final row is not allowed since 2008 would not be the shortest side. So six possible triangles in total.

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7. From 1 to 15 (and from $15n+1$ to $15(n+1)$ for any n) exactly 8 numbers will be said aloud. $2008=8\times250+8$ so by the time the game reaches $15\times250=3750$ then $8\times250=2000$ numbers will have been said aloud. The eighth number to be said aloud after this will be $3750+14=3764$

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8. $2008=251\times8=8+8+8+...+8+8+8+...8+8+8$

$$=(-117)+(-116)+(-115)+...+7+8+9+...+131+132+133$$

$$=118+119+...+132+133$$

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9. . N: it occurs 8260 times. E: 7327. Of the 18 letters used, L is rarest with only 40.

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10. $2008=2^3\times251$. Remove all the multiples of 2 (1004 of them) and all the multiples of 251 (8 of them) but then the 4 multiples of 2×251 have been removed twice. This gives $2008-(1004+8)-4=1000$ numbers co-prime with 2008.