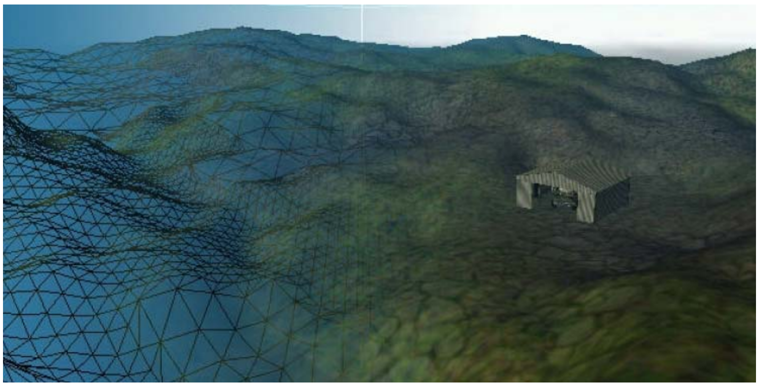
Vectors and Computer Graphics

Mathematics is used to make viewers believe that the computer generated worlds they are taken into in films are real. At the heart of this is the kind of geometry that you learn at school, that which deals with straight lines, angles and triangles.

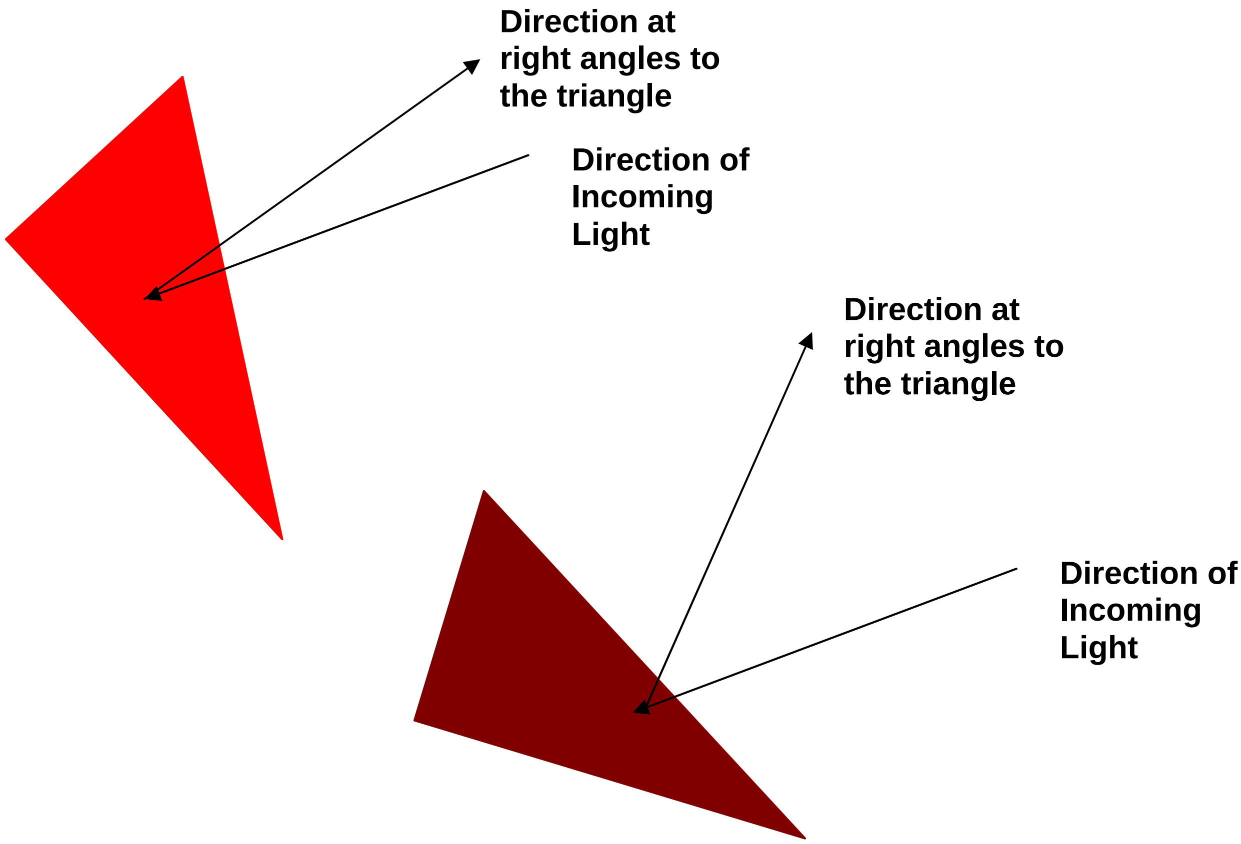


Modern video games use a high level of mathematics to simulate convincing 3D environments on-screen. This involves producing textures, adding shading and lighting, and using physics amongst many other thing. With computer generated imagery a polygon mesh is used to provide the framework for a 3D environment on-screen. This is shown in the image above. Each polygon must be filled with the correct texture.

These will be stored as standard textures and must be distorted as required using linear transformations or matrices. This is summed up in the diagram below.

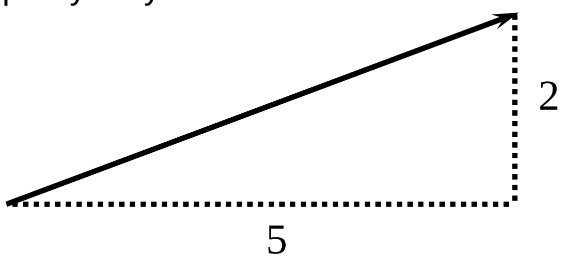
A scene must also be lit correctly according to the effects of the light sources in it. This is what gives a film its atmosphere. Compare the unlit and lit scenes below:

For a given polygon in a 3D environment, the angle between the normal to its plane and the light source will determine how bright that polygon should appear. This means that the calculation to determine the brightness of a polygon involves the scalar product.



How is this done?

First of all you need to know how vectors describe direction. In two dimensions this is pretty easy



$$\begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

In three dimensions the idea is the same. You can imagine that this three dimensional vector starts at the same point as the one above but it's end point is three units out of the page in your direction compared to the one above.

$$\begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$$

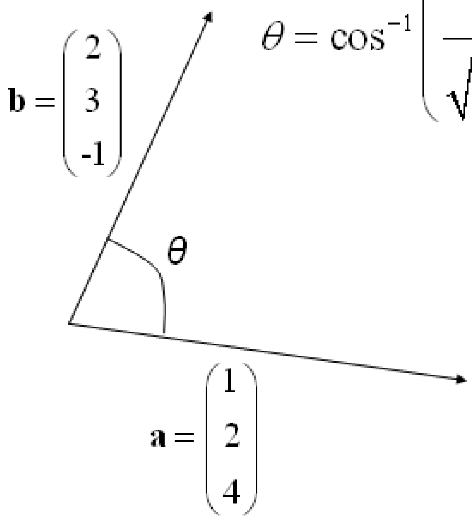
Exercise 1

Taking positive x to be right, positive y to be up and positive z be towards you, point in the following directions.

$$\begin{pmatrix}0\\0\\1\end{pmatrix} \qquad \begin{pmatrix}0\\-1\\0\end{pmatrix} \qquad \begin{pmatrix}1\\1\\4\end{pmatrix} \qquad \begin{pmatrix}-2\\3\\-6\end{pmatrix}$$

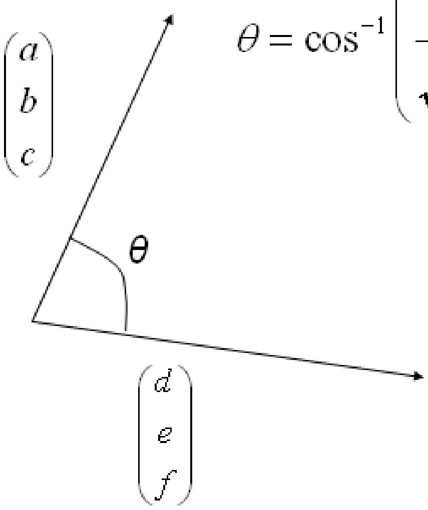
There is a relatively simple formula for calculating the angle between two vectors:

$$\theta = \cos^{-1}\left(\frac{(1\times 2)+(2\times 3)+(4\times -1)}{\sqrt{\left(1^2+2^2+4^2\right)\times\left(2^2+3^2+(-1)^2\right)}}\right)$$



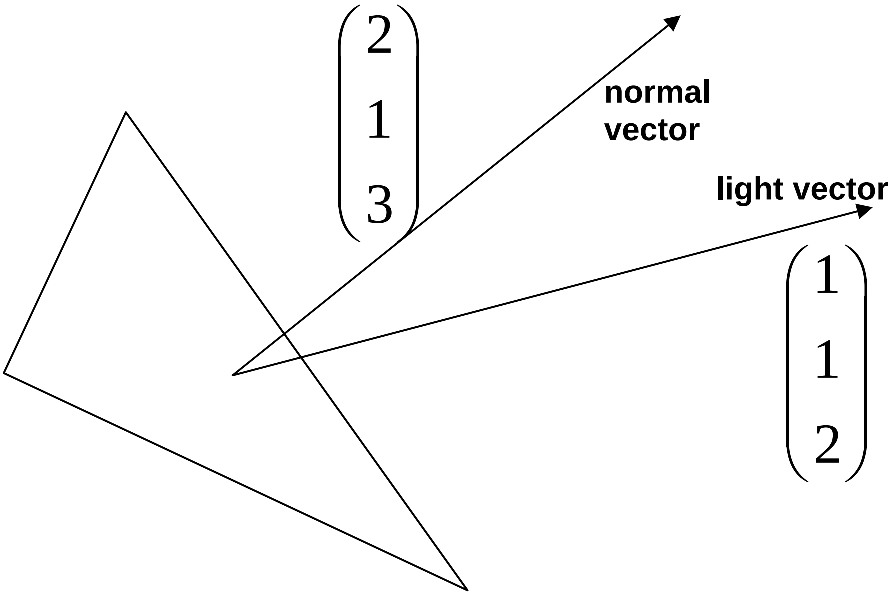
In general:

$$\theta = \cos^{-1}\left(\frac{(a\times d)+(b\times e)+(c\times f)}{\sqrt{\left(a^2+b^2+c^2\right)\times\left(d^2+e^2+f^2\right)}}\right)$$



Exercise 2

Calculate the angle between the light vector and the normal vector to the triangular polygon below. Hence find which colour the polygon should be shaded.



Angle	0° - 15°	15° - 30°	30° - 45°	45° – 60°	60° - 75°	75° - 90°
Colour	A	B	C	D	E	F

Answer to exercise 2:

$\theta = \cos^{-1}\left(\frac{9}{\sqrt{14}\sqrt{6}}\right) = 10.89^{o}$, so colour A.