

Introduction

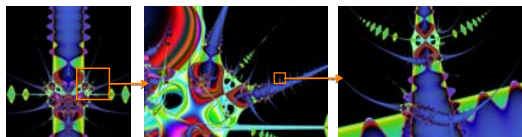
What is the equation of a straight line? How do you describe a circle in mathematical terms? No problem at all for a brilliant mathematical mind like yours: for example $y - 3 = 4(x - 2)$ or $x^2 + (y - 3)^2 = 9$.

But think of more complex shapes that occur in the physical world. How would you describe the path of lightning? Or the outline of a tree? What about coastlines, clouds, or even those intricate icy shapes that like to form on your car windows on a frosty night?

In fact, the vast majority of natural objects and phenomena in our world defy traditional mathematical description. However, a lot of them can be approximated by fractals, which brings us neatly onto our subject of interest...

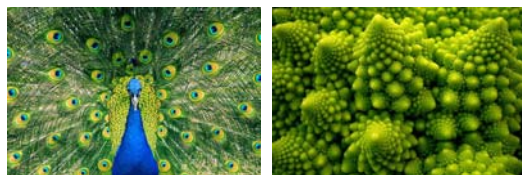
What are fractals?

A fractal is any geometric pattern that reveals greater and greater complexity as it is enlarged. The more you keep magnifying a fractal, the more details you discover.



Fractal zoom. If we had space we could keep zooming forever... We created this alien-like fractal with the free Sterling2 program.

You could say that, graphically, fractals portray the concept of 'worlds within worlds'. If you are one of many who struggle with the notion of infinity, or have ever wondered what infinity might look like, or just need an excuse to spend a few spare hours on YouTube, you can visually explore the infinity that is graphically manifest in fractals.



Peacock fractal (photo by William Branga)

Romanesque broccoli fractal (photo by lin.G)

Natural objects show the same patterns of symmetry and self-similarity, although their repetition of structure is not perfect. For example you can't infinitely zoom into a broccoli. Its structure is of course finite, but it can be modelled as a fractal.

How are fractals produced?

The simplest fractals are constructed by iteration, that is repeating a process over and over. The result of one iteration is used as the starting point for the next. There are three basic types of iteration that create fractals:

- **Iterated Function System:** repeatedly applies geometric transformations like rotations and reflections to points (see the 'Sierpinski's Carpet' example)
- **Generator Iteration:** starts with a geometric shape and substitutes parts of that shape with another figure, called the generator, then repeats this process
- **Formula Iteration:** repeatedly applies a mathematical formula, or several formulas, like the Newton-Raphson method

Fractal Art

The great thing about fractals is that you can create amazing art with them. As more and more people discover the beauty of fractals there are hundreds of websites devoted to their creation and to exploring different styles of fractal art.



Cats in a Bag by Roger Johnston

Shown here are two fractal artworks that have been created by Roger Johnston using the fractal flame editor Apophysis. (Fractal flames are a type of Iterated Function System). Roger has produced many stunning fractals with Apophysis. Some of his creations are for sale in a New York gallery & you can even buy T-shirts imprinted with his fractal designs.



Jelly Flame by Roger Johnston

The background of this poster is by Victoria Nece, a motion graphic artist also based in New York. She makes her fractals using AEFflame, a free plugin for Adobe After Effects.

Why not give it a go? No in-depth knowledge about fractals is required. Just a dose of creativity, and a computer of course. There is plenty of good freeware available on the net, such as Apophysis, AEFflame and Sterling2.

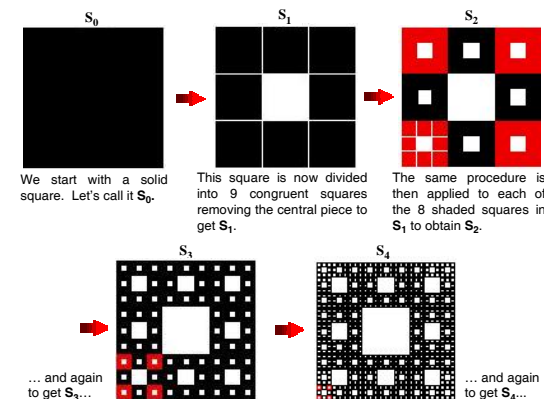
Further info

If you would like to find out more about fractals – whether you want to explore the concept of a 'visible' infinity, check out fractal art galleries, get a shiny new desktop background, or create your own fractal art – the internet is the place to go. We don't want to bore you with a long list of websites, just pop 'fractals' into a search engine and away you go.

You can view Roger's art online at community.webshots.com/user/rajahh and you can download Victoria's fractal wallpaper at victorianece.com. For more details about the information on this poster or a PDF copy please email anne.bergmann@students.plymouth.ac.uk.

Sierpinski's Carpet

Clear as mud so far? Let's look at Sierpinski's carpet, an example of an Iterated Function System.



Starting to see the pattern? Notice that with each iteration the side length of every square is enlarged with a scale factor $\frac{1}{3}$, centre at the bottom left of the original square, then translated to its new position. The rules of this entire iterative process are 8 simple linear transformations, let's denote them IF_1, IF_2, \dots, IF_8 . They can be expressed in matrix form as follows:

In each transformation IF_n the same matrix reduces the size of the square. The column vector $\begin{pmatrix} a(n) \\ b(n) \end{pmatrix}$ then shifts it.

$$IF_n = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a(n) \\ b(n) \end{pmatrix}$$

For IF_1, \dots, IF_8 the 8 column vectors are:

$$\begin{pmatrix} \frac{1}{3} \\ 0 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ 0 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{2}{3} \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix} \begin{pmatrix} 0 \\ \frac{2}{3} \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Let's look at the effect these transformations have on the area of S_0 . Assume that S_0 has side length of 1 unit, this means its area is 1 unit². Since the sides are scaled by $\frac{1}{3}$, the area of each new square is reduced to $(\frac{1}{3})^2$ of its size. So with every iteration each square turns into 8 squares that are $\frac{1}{3}$ its size. For example S_1 has $\frac{8}{9}$ the area of S_0 . In general, the area of S_{k+1} is $\frac{8}{9}$ of the area of S_k .

This means that the area of S_n is $(\frac{8}{9})^n$ for all natural numbers n . Notice that as n approaches infinity the area approaches a limiting value of 0 square units. In other words, in performing infinitely many iterations we have removed "all" of the area of our original square S_0 in constructing Sierpinski's carpet.

In fact, Sierpinski's carpet is just the set of points that remain after this construction is repeated infinitely many times, displaying that neat recurring pattern. Zoom in as much as you like on any part of Sierpinski's Carpet and you will see the same pattern again, and again, and again. **This is a fractal.**